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RF-3611

M. Sc. - II Examination

April / May - 2010

Mathematics : Paper - 5005

(Integral Transforms)

Time : **3** Hours] [Total Marks : **60**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **M. Sc. - 2**

Name of the Subject :

☛ **MATHEMATICS - 5005**

☛ Subject Code No. : **3 6 1 1** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Notations used are standard.

(4) Figure at the right end of the first line of each question indicate full marks.

Q.1	**(a)**	State and prove second shifting theorem for inverse Laplace transform.	[5]
	(b)	Using convolution theorem for Laplace transform,evaluate: $L^{-1}\left[\frac{1}{s(s^2+4)^2}\right]$	[5]
	(c)	Using Laplace Transform,evaluate : $\int_0^\infty \frac{1}{x^2}\sin^2(tx)dx$	[5]
		OR	
Q.1	**(d)**	In usual notations prove that: (i) $L[\frac{f(t)}{t}: s] = \int_s^\infty f(x)dx$ (ii) $L[\int_0^t f(u)du : s] = \frac{1}{s}\bar{f}(s)$	[5]
	(e)	Evaluate: $L^{-1}[\frac{1+2s}{(s+2)^2(s-1)^2}]$	[5]
	(f)	Using Laplace transform, show that: $\int_0^\infty e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$	[5]
Q.2	**(a)**	Define cosine integral and derive it's Laplace transform	[5]
	(b)	Using Laplace transform, Solve the integral equation: $f(t)= e^{-t} - 2\int_0^t \cos(t-u)F(u)du$	[5]
	(c)	Using Laplace transform solve the differential equation: $ty''+(2t+3)y'+(t+3)y=ae^{-t}$	[5]
		OR	

Q.2	**(d)**	Define exponential integral function and derive it's Laplace transform	[5]
	(e)	Find L[erf($\sqrt{t}$)] and hence deduce L[terf($2\sqrt{t}$)] = $\frac{3s+8}{s^2(s+4)^{3/2}}$	[5]
	(f)	Find $L^{-1}\left[\frac{e^{-\sqrt{s}}}{s}\right]$ and hence deduce that $L^{-1}\left[\frac{e^{-x\sqrt{s}}}{s}\right] = erfc(\frac{x}{2\sqrt{t}})$	[5]
Q.3	**(a)**	State and prove Fourier integral theorem	[5]
	(b)	Find Fourier of f(x)=$\begin{cases}1-x^2 & \text{for } \lvert x\rvert\le 1\\ 0 & \text{for } \lvert x\rvert>1\end{cases}$	[5]
	(c)	Show that $F_S[e^{-ax}:\xi] = \sqrt{\frac{2}{\pi}}\frac{\xi}{a^2+\xi^2}$	[5]
		OR	
Q.3	**(d)**	State and prove: (i) Linearity property for complex fourier transform. (ii)Change of scale property for Fourier sine transform.	[5]
	(e)	Find fourier sine and cosine transform of x^{m-1}	[5]
	(f)	State and prove Modulation theorem for Fourier transform	[5]
Q.4	**(a)**	State and prove convolution theorem for Fourier transform and derive Parsevals relation from it.	[5]
	(b)	Find $F_C^{-1}[\bar{f}_S(\xi)]$ where $\bar{f}_S(\xi) = \xi^n e^{-a\xi}$	[5]
	(c)	Using Parseval's identity prove that: $\int_0^\infty \frac{\sin(at)}{t(a^2+t^2)}dt = \frac{\pi}{2}\left[\frac{1-e^{-a^2}}{a^2}\right]$	[5]
		OR	
Q.4	**(d)**	Using Fourier integral formula prove that: $e^{-x}\cos(x) = \frac{2}{\pi}\int_0^\infty \frac{(u^2+2)\cos(ux)}{u^4+4}du$	[5]
	(e)	In usual notations prove that : $F_c^*[f''(x):p] = -p^2 f_c^*(p) - f'(0) + (-1)^p f''(\pi)$ and hence derive formula for $F_c^*[f^{(4)}(x):p]$	[5]
	(f)	Evaluate $F_c^*[f(x):p]$ where f(x) = 1 for $x \in (0,\frac{\pi}{2})$ and f(x) = -1 for $x \in (-\frac{\pi}{2},\pi)$	[5]
Q.5	**(a)**	Using Laplace transform solve: $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} - xt = 0$ if $u(0)=u_t(0)=0$	[6]
	(b)	The temperature V in a semi-infinite rod $0 \le x < \infty$ is determined by $\frac{\partial V}{\partial t} = k\frac{\partial^2 V}{\partial x^2}$; $x \in (0,6)$, $t > 0$ Subject to the conditions: (i) u=0 when t=0, x>0, (ii) $\frac{\partial V}{\partial x} = \mu$ When x=0 and (iii) $\frac{\partial V}{\partial x} \to 0$ as $x \to \infty$ Determine the temperature formula.	[8]
		OR	
Q.5	**(c)**	Using Laplace transform solve : $[tD^2 + D + t]y = 0$; y(0)=1	[6]
	(d)	Usng finite Fourier transform solve: $\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}$; given that V(0,t)=0; V(π,t)=0; V(x,0)=2x, where $0<x<\pi$; $t>0$	[8]